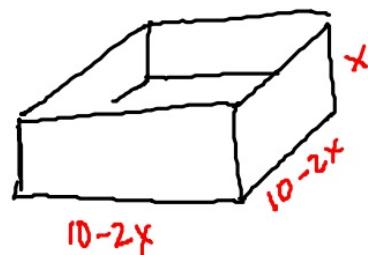
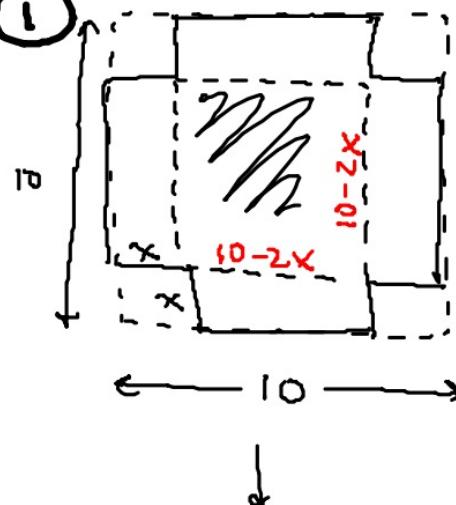


OPTIMIZATION W.S

1



$$V = x(10-2x)^2$$

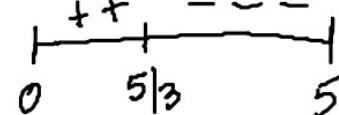
$$V' = (10-2x)^2 + x \cdot 2(10-2x)(-2) = 0$$

$$(10-2x)(10-2x-4x) = 0$$

$$(10-2x)(10-6x) = 0$$

$$x = 5, \frac{5}{3}$$

CONSIDER DOMAIN: $x \in (0, 5)$



since V' changes from $+ \infty$ to $-$,
max at $x = \frac{5}{3}$.

THE SQUARES SHOULD
BE $\frac{5}{3}'' \times \frac{5}{3}''$.

THE MAX. VOLUME

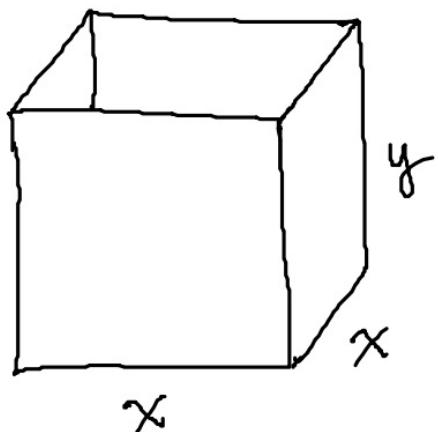
$$\text{IS } V\left(\frac{5}{3}\right)$$

$$= \frac{5}{3} \left(\frac{10}{3} - \frac{10}{3}\right)^2$$

$$= \frac{5}{3} \left(\frac{20}{3}\right)^2 = \frac{5}{3} \cdot \frac{400}{9}$$

$$= \frac{2000}{27} \text{ m}^3.$$

(3)



$$x^2 + 4xy = 108$$

$$4xy = 108 - x^2$$

$$y = \frac{108 - x^2}{4x}$$

$$y(6) = \frac{108 - 36}{24} = 3$$

$$V = x^2 y$$

$$V = x^2 \left(\frac{108 - x^2}{4x} \right)$$

$$V = \frac{1}{4} (108x - x^3)$$

$$V' = \frac{1}{4} (108 - 3x^2) = 0$$

$$108 = 3x^2$$

$$36 = x^2$$

$$x = \cancel{-6}, 6$$

$$\text{DOMAIN: } x \in (0, \sqrt{108})$$

$$V'' = \frac{1}{4} (-6x)$$

$$V''(6) = \frac{1}{4} (-36) < 0$$

$\therefore \text{MAX AT } x = 6.$

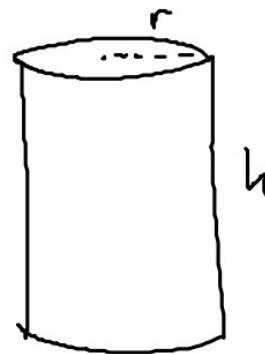
THE DIMENSIONS ARE

$$6'' \times 6'' \times 3''$$

Recipe for Optimization Problems:

1. Make a drawing - label it.
2. Create a formula/equation for the quantity you are trying to optimize.
3. Use the initial condition (secondary equation) to make a substitution (if necessary.)
4. Differentiate and find critical point(s).
5. Verify the max/min on an appropriate **domain** (first or second deriv. test.)
6. Answer the question!

(10)



$$\pi r^2 h = 750 \text{ cm}^3$$

$$h = \frac{750}{\pi r^2}$$



$$SA = 2\pi r^2 + 2\pi r h$$

$$= 2\pi r^2 + 2\pi r \cdot \frac{750}{\pi r^2}$$

$$= 2\pi r^2 + 1500r^{-1}$$

$$SA' = \left(4\pi r - \frac{1500}{r^2} = 0 \right) r^2$$

$$4\pi r^3 - 1500 = 0$$

$$r = \sqrt[3]{\frac{1500}{4\pi}} \approx 4.924 \text{ cm}$$

$$r \in (0, \infty)$$

$$SA'' = 4\pi + 1500r^{-3}$$

$$SA'' \left(\sqrt[3]{\frac{1500}{4\pi}} \right) = 4\pi + \frac{1500}{\frac{1500}{4\pi}}$$

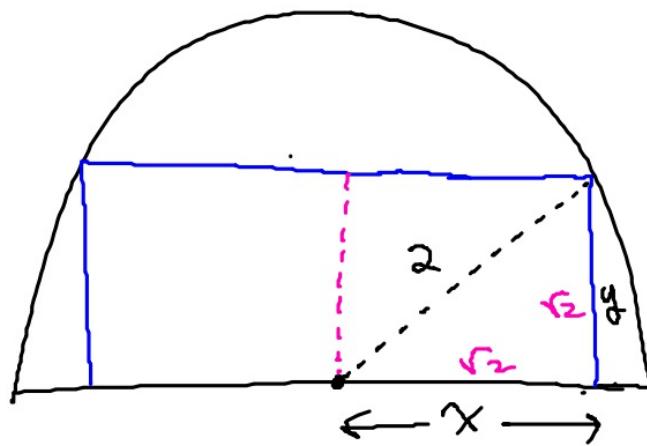
$$> 0$$

\therefore min at $r = 4.924$.

$$h \left(\sqrt[3]{\frac{1500}{4\pi}} \right) \approx \underline{\underline{9.847 \text{ cm}}}$$

THE RADIUS IS 4.924 cm,
THE HEIGHT IS 9.847 cm.

2



$$x^2 + y^2 = 4$$

$$y^2 = 4 - x^2$$

$$y = \sqrt{4 - x^2}$$

$$y(\sqrt{2}) = \sqrt{4 - 2} = \sqrt{2}$$

$$A = 2xy$$

$$A = 2x(\sqrt{4-x^2})$$

MAXIMIZE A^2 INSTEAD

$$A^2 = 4x^2(4-x^2)$$

$$A^2 = 16x^2 - 4x^4$$

$$(A^2)' = 32x - 16x^3 = 0$$

$$16x(2 - x^2) = 0$$

$$x = \sqrt{2}$$

DOMAIN: $x \in (0, 2)$

$$(A^2)'' = 32 - 48x^2$$

$$\begin{aligned} (A^2)(\sqrt{2}) &= 32 - 48\sqrt{2} \\ &= 32 - 96 < 0 \end{aligned} \quad \therefore \text{max}$$

THE DIMENSIONS

ARE $2\sqrt{2}$ BY $\sqrt{2}$.